Why Teach
Mathematics with Manipulatives?
ETA hand2mind® Research Summary

Sara Delano Moore, Ph.D.

I hear and I forget.
I see and I remember.
I do and I understand.
—Confucius (551–479 BC)

Overview

The use of manipulatives in teaching mathematics has a long tradition and solid research history. Manipulatives not only allow students to construct their own cognitive models for abstract mathematical ideas and processes, they also provide a common language with which to communicate these models to the teacher and other students.

In addition to the ability of manipulatives to aid directly in the cognitive process, manipulatives have the additional advantage of engaging students and increasing both interest in and enjoyment of mathematics. Students who are presented with the opportunity to use manipulatives report that they are more interested in mathematics. Long-term interest in mathematics translates to increased mathematical ability (Sutton & Krueger, 2002).

"[I]n order to develop every student's mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels."
—NCSM, 2013

In 2013, the National Council of Supervisors of Mathematics (NCSM) issued a position statement on the use of manipulatives in classroom instruction to improve student achievement. "[I]n order to develop every student's mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels." (NCSM, 2013)] This position is based on research supporting the use of manipulatives in classroom instruction. For example, Ruzic & O'Connell (2001) found that long-term use of manipulatives has a positive effect on student achievement by allowing students to use concrete objects to observe, model, and internalize abstract concepts.

Description and History

Manipulatives are concrete objects that can be viewed and physically handled by students in order to demonstrate or model abstract concepts. They represent a category of mathematical tools that are referenced in mathematics standards such as the Mathematics Process Standards included in *Principles and Standards for School Mathematics* (NCTM, 2000) or the Standards for Mathematical Practice included in the *Common Core State Standards for Mathematics* (National Governor's Association et al, 2010).

John van de Walle and his colleagues (2013) define a mathematical tool as, "any object, picture, or drawing that represents a concept or onto which the relationship for that concept can be imposed. Manipulatives are physical objects that students and teachers can use to illustrate and discover mathematical concepts, whether made specifically for mathematics (e.g., connecting cubes) or for other purposes (e.g., buttons)" (p 24). More recently, virtual manipulative tools are available for use in the classroom as well; these are treated in this document as a tool for teacher modeling and demonstration.

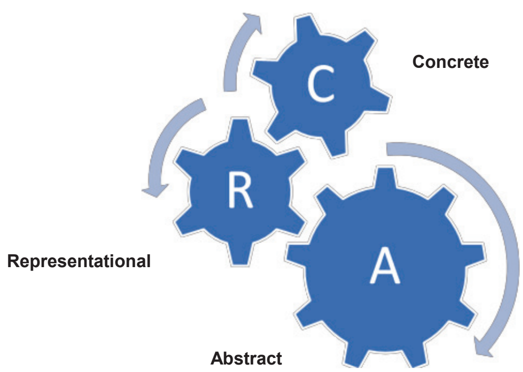
The history of manipulatives for teaching mathematics extends at least two hundred years. More recent important influences have included Maria Montessori (1870–1952), Jean Piaget (1896–1980), Zoltan Dienes (1916–), and Jerome Bruner (1915–). Each of these innovators and researchers has emphasized the importance of authentic learning experiences and the use of concrete tools as an important stage in development of understanding. Piaget (1952) suggests that children begin to understand symbols and abstract concepts only after experiencing the ideas on a concrete level. Dienes (1960) extended this to suggest that children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics. Their pioneering work has led to many studies of the importance of manipulatives for student learning in mathematics.

Support from Research

Researchers over the past forty years have generally found that manipulatives are a powerful addition to mathematics instruction. Meta-analyses by Suydam & Higgins (1977), Parham (1983), and Sowell (1989) found that achievement in mathematics could be increased by the long-term use of manipulatives. Well over 100 studies are included among these three analyses.

Three more recent publications confirm similar findings. In 2001, the National Research Council released the book *Adding It Up: Helping Children Learn Mathematics*. This book reflects a comprehensive review of mathematics education research and includes this statement:

> "The evidence indicates, in short, that manipulatives can provide valuable support for student learning when teachers interact over time with the students to help them build links between the object, the symbol, and the mathematical idea both represent." (p 354)



The 2009 What Works Clearinghouse report *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* includes using visual models and manipulatives as one of its research-based recommendations. In addition, Appendix D of the report summarizes research supporting the use of the Concrete–Representational–Abstract (CRA) approach for helping students make connections between concrete manipulatives and abstract mathematical ideas. In this approach, built on Bruner's (1966) work, students first use concrete materials to solve problems and look for patterns and generalizations. As students need to record their work, they do so first by sketching pictures (representations) of the manipulative models and then finally move to using abstract (and more formal) mathematical notations for their work. Moore (2013) provides an example of this structure in action.

Wenglinsky's (2000) analysis of NAEP data suggests the value of interaction over time, another key idea from *Adding It Up*. By examining data about classroom activity, this analysis suggests that, "when students are exposed to hands-on learning on a weekly rather than a monthly basis, they prove to be 72% of a grade level ahead in mathematics" (p 27). Not only is it essential to help students make the connections between concrete manipulatives and abstract mathematics, but it is also essential that this instructional strategy be a frequent element of classroom practice.

Assessment for Learning

Manipulatives are a powerful tool for supporting classroom assessment. The literature review by Black & Wiliam (1998) found that formative assessment practices yielded effect sizes of one-half to one full standard deviation. More importantly, "improved formative assessment helps low achievers more than other students and so reduces the range of achievement while raising achievement overall." (p. 141) In other words, formative assessment increases student learning substantially while reducing achievement gaps. There are many definitions of formative assessment. Stiggins' (2005) description suggests the critical element – teachers use classroom assessment to make changes in instruction in real time to result in real increases in learning.

> Another difference is that traditional formative thinking tends to want more frequent assessment of student mastery of the standards themselves, while assessment FOR learning focuses on day-to-day progress in learning as students climb the curricular scaffolding leading up to state standards. It tells users if and when students are attaining the foundations of knowledge, the reasoning, the performance skills, and the product development capabilities that underpin the mastery of essential standards. (p. 328)

In order to implement formative assessment well, Wiliam (2011) shares five key strategies. Three are particularly well-supported by manipulative-based instruction: classroom activities and learning tasks which elicit evidence of learning, activating learners as instructional resources for one another, and activating learners as owners of their own learning. In all three cases, by actively engaging students in the doing of mathematics, manipulatives provide a foundation which encourages discussion and student ownership of their work. This provides teachers with a vivid current picture of student understanding and guides teachers in determining appropriate next steps.

Summary

Building on the learning theory work of Piaget and Bruner, a solid history of research supports the regular use of manipulatives in classroom mathematics instruction. While children can remember, for short periods of time, information taught through books and lectures, deep understanding and the ability to apply learning to new situations requires conceptual understanding that is grounded in direct experience with concrete objects. It is also important to note the critical role of the teacher in helping students connect their manipulative experiences, through a variety of representations, to essential abstract mathematics. Together, excellent teachers and regular experiences with hands-on learning can provide students with powerful learning in mathematics.

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