“Algebra for All” and “No Child Left Behind” are phrases in the education community that suggest, and in many cases require, action. They give impetus for mathematics teachers at all levels to provide instruction that leads students to deeper understanding of algebraic concepts. Additionally, recent legislation in some states requires that special needs students must also complete coursework in algebra. Hands-on materials have been shown to be effective in aiding that instruction.

Algebra Tiles are a versatile manipulative that may be used by students for pre-algebra concepts up through factoring and equation solving. Working with Algebra Tiles provides activities that focus on working with Algebra Tiles provides activities that focus on working with Algebra Tiles through factoring and equation solving. Working with Algebra Tiles provides activities that focus on working with Algebra Tiles through factoring and equation solving.

CONTENTS

Unit 1: Introduction to Algebra Tiles .......................................................... 1–9
Overview and Answers ........................................................................... 2–3
Activity 1.1 - Integer Pieces ................................................................. 4
Activity 1.2 - The Zero Principle ............................................................ 5
Activity 1.3 - Modeling Integers .............................................................. 6
Activity 1.4 - Naming Algebra Tile Pieces ........................................... 7–8
Activity 1.5 - Grab Game .................................................................... 9–10
Unit 2: Integer Operations .................................................................... 11–32
Overview and Answers ........................................................................... 12–15
Activity 2.1 - Modeling Addition ......................................................... 16
Activity 2.2 - Addition of Integers ......................................................... 17–18
Activity 2.3 - Three Addends ................................................................. 19
Activity 2.4 - The Game of ONE .......................................................... 20–21
Activity 2.5 - Modeling Subtraction ..................................................... 22–23
Activity 2.6 - Subtraction and Addition Patterns .................................. 24
Activity 2.7 - Subtraction of Integers .................................................... 25
Activity 2.8 - Least is Best .................................................................. 26–28
Activity 2.9 - Modeling Multiplication ................................................ 29–30
Activity 2.10 - Multiplication of Integers ............................................. 31–32
Unit 3: Linear Expressions .................................................................... 33–57
Overview and Answers ........................................................................... 34–36
Activity 3.1 - Translating Algebraic Expressions ................................... 37
Activity 3.2 - Spin an Expression .......................................................... 38–39
Activity 3.3 - Expression Match ............................................................ 40–46
Activity 3.4 - Evaluating Linear Expressions ....................................... 47–48
Activity 3.5 - Closest to Zero ................................................................. 49
Activity 3.6 - Distributive Property ....................................................... 50
Activity 3.7 - Adding Linear Expressions ............................................. 51–52
Activity 3.8 - Find the Sum: Adding Linear Expressions ..................... 53–56
Activity 3.9 - Subtracting Linear Expressions ...................................... 57
Unit 4: Linear Expressions .................................................................... 59–65
Overview and Answers ........................................................................... 60–61
Activity 4.1 - Solving Linear Equations (a = 1) .................................... 62
Activity 4.2 - Solving Linear Equations (a = -1) ................................... 63
Activity 4.3 - Solving Linear Equations with Variables on Both Sides ........................................................................ 64
Activity 4.4 - Solving Linear Inequalities ............................................. 65
Unit 5: Linear Expressions .................................................................... 67–98
Overview and Answers ........................................................................... 68–71
Activity 5.1 - Modeling Quadratic Expressions in Two Variables ........ 72
Activity 5.2 - Evaluating Quadratic Expressions .................................. 73–74
Activity 5.3 - Adding and Subtracting Quadratic Expressions ........... 75–76
Activity 5.4 - Multiplying Binomials .................................................... 77–78
Activity 5.5 - Multiplying Binomials Involving (x – c) ......................... 79–80
Activity 5.6 - Product Match Games .................................................... 81–94
Activity 5.7 - Making Squares ............................................................... 95–96
Activity 5.8 - Making Rectangles ........................................................ 97
Activity 5.9 - Factoring x² + bx + c ....................................................... 98
Unit 6: Problem Solving ........................................................................ 99–110
Overview and Answers ........................................................................... 100–101
Activity 6.1 - Perimeter Problem Solving ............................................. 102–103
Activity 6.2 - Solving Number Puzzles with Algebra Tiles ............... 104–105
Activity 6.3 - Will It Factor? ................................................................. 106–108
Activity 6.4 - Algebra Array ................................................................. 109–110
ALGEBRA STANDARD

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

<table>
<thead>
<tr>
<th>Understand patterns, relations and functions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grades 6-8</strong></td>
<td><strong>Grades 9-12</strong></td>
</tr>
<tr>
<td>Represent, analyze and generalize a variety of patterns with tables, graphs, words and, when possible, symbolic rules;</td>
<td>Understand relations and functions and select, convert flexibly among, and use various representations for them;</td>
</tr>
<tr>
<td>Relate and compare different forms of representation for a relationship.</td>
<td>Understand and perform transformations such as arithmetically combining, composing and inverting commonly used functions using technology to perform such operations on more complicated symbolic expressions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Represent and analyze mathematical situations and structures using algebraic symbols</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grades 6-8</strong></td>
<td><strong>Grades 9-12</strong></td>
</tr>
<tr>
<td>Develop an initial conceptual understanding of different uses of variables;</td>
<td>Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;</td>
</tr>
<tr>
<td>Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;</td>
<td>Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;</td>
</tr>
<tr>
<td>Recognize and generate equivalent forms for simple algebraic expressions and solve linear relationships.</td>
<td>Use symbolic algebra to represent and explain mathematical relationships.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Use mathematical models to represent and understand quantitative relationships</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grades 6-8</strong></td>
<td><strong>Grades 9-12</strong></td>
</tr>
<tr>
<td>Model and solve contextualized problems using various representations, such as graphs, tables, and equations.</td>
<td>Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.</td>
</tr>
</tbody>
</table>

In having students work with Algebra Tiles, teachers need to realize that there will be an initial investment of classroom time that will pay dividends later as students continue their study of algebra. The models constructed for work with integers are extendable for other algebraic concepts; therefore, additional classroom time will not be needed as students use Algebra Tiles to explore these concepts.
Several of the units provide activities for students to express or conjecture in writing their mathematical thinking about specific algebraic concepts. These writing opportunities are at the heart of the NCTM Communication Standard.

**COMMUNICATION STANDARD**

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

<table>
<thead>
<tr>
<th>Organize and consolidate their mathematical thinking through communication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communicate their mathematical thinking coherently and clearly to peers, teachers, and others</td>
</tr>
<tr>
<td>Analyze and evaluate the mathematical thinking and strategies of others</td>
</tr>
<tr>
<td>Use the language of mathematics to express mathematical ideas precisely</td>
</tr>
</tbody>
</table>

Modeling, computing, simplifying, solving, writing, and conjecturing are just a few of the tasks that are critical to understanding algebra. These suggest our students be active in their learning. *Working with Algebra Tiles* provides these opportunities.
INTRODUCTION TO ALGEBRA TILES

Unit 1
UNIT 1  Introduction to Algebra Tiles

Unit Overview
Areas of squares and rectangles are key to naming and working with Algebra Tile pieces. Beginning with the small yellow/red squares, it is common to denote the length of each side as 1. Therefore, the area is 1. The value of a yellow square is 1, whereas the value of a red square is −1. The blue/red square customarily has sides of length x. Its area is $x^2$. The value of a blue square is $x^2$, whereas the value of a red square is $-x^2$. The dimensions of a green/red rectangle are 1 unit by x units, and its area is $x$. Activities in the unit first involve representing integers using the small squares, and then proceed to representing and writing algebraic expressions using the Algebra Tiles. The Zero Principle is introduced and used throughout this unit and future units.

Glossary of Terms
Integers: The set of numbers . . . −3, −2, −1, 0, 1, 2, 3, . . .
Opposites: Two numbers that have the same absolute value but opposite signs.
Zero Pair: Two tiles, one of each color. A zero pair sums to zero.
Variable: A symbol, usually a letter, that is used to represent one or more numbers in an algebraic expression. For example, x is a variable in the expression $4x + 3$.
Polynomial: An expression that has one or more terms of the form $ax^n$ where $a$ is any real number and $n$ is a whole number.
Binomial: A polynomial that has two terms.
Trinomial: A polynomial that has three terms.
Term: A part of an expression that is separated by an addition or subtraction sign. In the expression $2x − 4$, the terms are $2x$ and $−4$.

Activity 1.1 Integer Pieces
Objective: To represent positive and negative integers using Algebra Tiles.
Prerequisites: Students need to know that the area of a square is given by $A = s^2$, where s is the length of a side.
Getting Started: Introduce the unit squares. Discuss the idea of unit length, so that the area of the square is 1. Discuss the idea of a negative integer. Note that we can use the yellow unit squares to represent positive integers and the red unit squares to represent negative integers.
Closing the Activity: Present verbal descriptions to students and have them model the result with Algebra Tiles.
Answers:
1. 8 yellow pieces
2. 3 red pieces
3. Answers vary
4. 2 yellow pieces
5. 10 red pieces
6. 5 red pieces
7. (a) −4 (b) 9 (c) 2
8. Possible answer: make a zero pair with one yellow piece and one red piece. Two yellow pieces remain. Answer is 2.

Activity 1.2 The Zero Principle
Objective: To represent the opposite of an integer using Algebra Tiles. To represent zero pairs using Algebra Tiles.
Prerequisites: Students must be able to represent positive and negative integers using Algebra Tiles.
Getting Started: Show students how two small squares of opposite colors neutralize each other, so that the net result of such a pair is zero.
Closing the Activity: Present various collections of unit squares, positive and negative, and have students use the Zero Principle to simplify the collection. Given that there are an odd number of unit squares, have students determine possible outcomes: all yellow, positive integer; all red, negative, integer; more yellow than red, positive integer; more red than yellow, negative integer.
Answers:
1. (a) $5 + (-5) = 0$  (b) $3 + (-3) = 0$  (c) $6 + (-6) = 0$
2. Answers vary
3. (a) $4 + (-4) = 0$  (b) $2 + (-2) = 0$  (c) $6 + (-6) = 0$
Activity 1.3 Modeling Integers

Objective: To model integers using zero pairs.
Prerequisites: Students need to be able to represent positive and negative integers using Algebra Tiles.

Getting Started: A positive integer can be represented by other positive integers in many ways. For example, $6 = 3 + 3 = 5 + 1 = 4 + 2$. Similarly, positive and negative integers can be represented by other positive and negative integers in many ways. Start with a positive integer such as 2. Add zero pairs and discuss the new name for 2. Continue this process several times. Repeat the activity; however, begin with a negative integer and add zero pairs.

Closing the Activity: Have students sketch more than one model for a given positive or negative integer. Give students an illustration of several positive and negative integers using Algebra Tiles, and have them name the integer represented in the model.

Answers:
1. (a) 2 (b) –6 (c) 4 (d) 3 (e) –4 (f) 1
2. (a)–(f) Teacher check
3. Infinite. Starting with 4 yellow tiles, as many zero pairs can be added as you wish.

Activity 1.4 Naming Algebra Tile Pieces

Objective: To name the remaining Algebra Tile pieces ($x$, $x^2$).

Prerequisites: An area model is used in naming Algebra Tile pieces. Students must know that the area of a square is given by $A = s^2$, where $s$ is the length of a side. Students must know that the area of a rectangle is given by $A = lw$, where $l$ is the length and $w$ is the width of the rectangle.

Getting Started: On an overhead projector, show the other Algebra Tile pieces. Show that the length of a side of the blue/red square is not an integer value. We will say that the length is $x$. Therefore, the area is $x^2$. The blue square has a value of $x^2$, whereas the red square has a value of $-x^2$. The green Algebra Tile piece has a value of $x$, and the red Algebra Tile piece has a value of $-x$. Place a collection of Algebra Tiles on an overhead projector and write an algebraic expression for the collection. Then write an algebraic expression and have students model it with the tiles.

Closing the Activity: The activity concludes with change in the length of a side of the small square. With a side of length $y$, the area now becomes $y^2$. The value of a yellow square is $y^2$, and the value of a small red square is $-y^2$. Expressions involving two variables can be written.

Answers:
1.–4. Teacher check
5. $x^2+3xy + 4y^2$
6. $2x^2 + (-2xy) + y^2$
7. $-3x^2 + 4xy + (-3y^2)$
8. 4 blue $a^2$ pieces, 2 red $ab$ pieces, 2 yellow $b^2$ pieces

Activity 1.5 Grab Game

Objective: To practice writing and simplifying algebraic expressions, using the Zero Principle.

Prerequisites: Students must know how to apply the Zero Principle in order to simplify algebraic expressions.

Getting Started: Using overhead Algebra Tiles, illustrate how to simplify an algebraic expression, using the Zero Principle where necessary.

Closing the Activity: Show illustrations of Algebra Tile pieces and have students write a simplified expression. Write algebraic expressions and have students write simplified expressions.
**Activity 1.1 Integer Pieces**

To represent integer values, use the small square tiles.

<table>
<thead>
<tr>
<th>Each white piece represents +1</th>
<th>Each gray piece represents −1</th>
</tr>
</thead>
</table>

Use the small square tiles to represent each number. Make a sketch of your model.

1. 8
2. −3
3. A single digit odd number
4. An even prime number
5. 10 feet below sea level
6. 5 degrees below zero

7. (a) (b) (c)

8. Explain how you determined the value represented in 7c.
ACTIVITY 1.2 The Zero Principle

In each example above, the two actions balance, or neutralize, one another.

Think of additional examples where two actions neutralize one another.

- ____________________________
- ____________________________
- ____________________________
- ____________________________

A small white square tile represents 1. A small gray square tile represents –1. The numbers 1 and –1 are opposites. They sum to 0. We refer to 1 and –1 as a zero pair.

1. Write an equation for each diagram.

2. Draw diagrams on the mats below that each have a value of 0. Write an equation for each.

Although it is not necessary, it is easier to read the value represented on the mat when the tiles have been arranged as shown.

3. Write an equation for each diagram.
ACTIVITY 1.3  
**Modeling Integers**

The integer 3 is modeled in each diagram below.

If the two zero pairs were removed, three white tiles would remain, or positive 3.

If the three zero pairs were removed, three white tiles would remain, or positive 3.

The integer –2 is modeled in the diagram below.

If the three zero pairs were removed two gray tiles would remain, or –2.

1. Name the integer modeled in each of the following.

(a) (d)  
(b) (e)  
(c) (f)

2. Sketch a model for each integer using white and gray tiles in each diagram.

(a) 4  (d) 5  
(b) –3  (e) –6  
(c) –1  (f) 2

3. How many different models are there for the integer 4? Explain.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
A variable, usually represented by a letter, is used to represent one or more numbers. If we let the variable $x$ represent the length of the large blue square tile, then $x^2$ represents the area of that tile.

We name the Algebra Tile pieces by considering the area of each, meaning we need to know the dimensions of each piece. If the small yellow square has length 1 unit, the area of the small square is 1 square unit. Then the long green rectangle has dimensions 1 by $x$ and has an area of $x$ square units.

We can use a collection of Algebra Tiles to model polynomials. The polynomial $2x^2 + x + 3$ is modeled below.

Use the Algebra Tiles to represent each polynomial. Make a sketch of your model.

1. $x^2 + 4x + 2$

2. $3x^2 + 2x$

3. A binomial

4. A trinomial
**ACTIVITY 1.4 Naming Algebra Tile Pieces**

The Algebra Tile pieces can be named using two different variables for the dimensions.

This is the \( x^2 \) piece.  
This is the \( xy \) piece.  
This is the \( y^2 \) piece.

Negative signs can be assigned to each shaded tile to represent negative terms in a polynomial.

This is the \(-x^2\) piece.  
This is the \(-xy\) piece.  
This is the \(-y^2\) piece.

Write the polynomial represented in each of the following.

5. ______________

6. ______________

7. ______________

8. Explain how you would name the Algebra Tile pieces in order to represent the polynomial  
\[ 4a^2 - 2ab + 2b^2 \].  
______________
This activity is for two students. Place a set of Algebra Tiles in a paper bag. Use the work mat on the next page. Take turns grabbing a set of Algebra Tiles from the bag and dropping them on your partner’s work mat. That student writes an expression for the Algebra Tiles grabbed on the appropriate blanks shown and then simplifies the expression. Here is an example:

\[ x^2 + (-x^2) + x^2 + x + (-x) + (-x) + (-x) + 1 + 1 \]

Using the Zero Principle, we can simplify this expression to \( x^2 - 2x + 2 \).

<table>
<thead>
<tr>
<th>Expression Grab</th>
<th>Example</th>
<th>( x^2 + (-x^2) + x^2 + x + (-x) + (-x) + (-x) + 1 + 1 = x^2 - 2x + 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expression 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expression 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expression 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expression 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expression 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, check all five of your expressions.

What expression has the most positive \( x \) terms? ..................................................

What expression has the fewest number of negative \( x^2 \) terms? ..................................

If there are any binomial expressions, write them below. (You may not have any!)