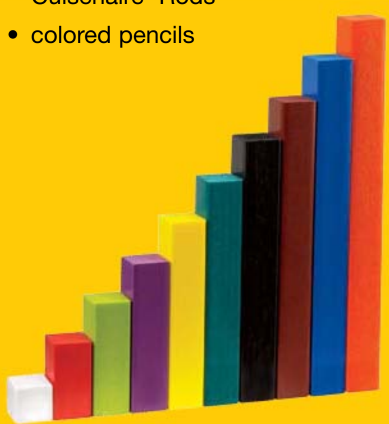
Common Core State Standard

4.NF.B.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

Materials

- Cuisenaire® Rods
- colored pencils



Decomposing Fractions

Students use Cuisenaire® Rods to show that a non-unit fraction can be decomposed into sums. In this activity, students will—

- use reasoning to model fractions decomposed into sums,
- develop their sense of composition of proper fractions, improper fractions, and mixed numbers, and
- generate sums of fractions.

Investigate

Have students explore the following problem.

Using Cuisenaire Rods, how many different ways can you show $1\frac{3}{4}$*?*

Let students consider the problem. Interact using prompts. For example, ask students which rods they might use to represent the whole. Ask if a brown rod would work as the whole. Elicit that a brown rod will work.

Verify that students are on track. For example, ask—

- Which rods work as the whole if you want to be able to show fourths? [brown, purple] Why?
- If 1 is brown, which color represents $\frac{1}{4}$? [red]
- Can you build a train to model $1\frac{3}{4}$ using brown and red rods? How?
- How can you express the length as a sum? [$1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$]

Have students build and sketch the model. Have them label each fractional part.

Ask students if this model is the only way to build $1\frac{3}{4}$ using a brown rod as the whole. Elicit that it is not, that there is a way to represent $\frac{3}{4}$ using one rod. Have students build the new model for $1\frac{3}{4}$. They will use a brown rod and a dark green rod. They should sketch the model they build and label each fractional part.

Ask students if they can write the sum for the train. [$1 + \frac{3}{4}$]

Prompt students further. You might ask—

- Have you found every way of modeling $1\frac{3}{4}$ using a brown rod as the whole?
- Can you find another way to build the $\frac{3}{4}$ part of your model?

Elicit that $\frac{3}{4}$ can be decomposed in more ways. For example—

This model can be described using the sum $1 + \frac{2}{4} + \frac{1}{4}$.

Ask students if there are still more ways to model $1\frac{3}{4}$—ways that can be described using different sums. Ask—

- How can you model $1\frac{3}{4}$ as a sum of fourths only?

- What sum describes the model? [$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$]

Let students continue; there are more ways to model $1\frac{3}{4}$. Remind students also that another rod besides the brown rod might work for the whole (but both cannot represent 1 at the same time).

Think and Share

Ask students to share their sums and some of the models they built. Write the sums on the board. The complete list is:

Brown = 1, Red = $\frac{1}{4}$
(or Purple = 1, White = $\frac{1}{4}$)
$1 \text{ (or } \frac{4}{4}) + \frac{3}{4}$
$1 \text{ (or } \frac{4}{4}) + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
$1 \text{ (or } \frac{4}{4}) + \frac{2}{4} + \frac{1}{4}$
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
$\frac{5}{4} + \frac{2}{4}$
$\frac{5}{4} + \frac{1}{4} + \frac{1}{4}$
$\frac{3}{4} + \frac{3}{4} + \frac{1}{4}$
$\frac{3}{4} + \frac{2}{4} + \frac{1}{4} + \frac{1}{4}$
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$\frac{2}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
Purple = 1, White = $\frac{1}{4}$
$\frac{6}{4} + \frac{1}{4}$
$\frac{7}{4}$

Use prompts such as these to promote class discussion:

- Why does the brown rod work as the whole? The purple rod?
- How did you know that you had found all sums with a denominator of 4?
- What if you consider the equivalent number $1\frac{6}{8}$? How many more sums could you find?

Teacher Talk

In the activity, students use their fraction sense to express a number in different ways. Upon first consideration of the task, a student will likely interpret $1\frac{3}{4}$ just as it is written—as a whole plus a fraction of that whole. This is a good starting point. Visually, mixed-number form is tied strongly to the number line, which students are most familiar with as a sequence of whole-number intervals that are themselves subdivided into unit-fraction intervals.

Upon further consideration, students realize that—

- the fraction part of the number can be decomposed and expressed as a sum of smaller fractions, and there might be more than one valid decomposition; and
- the whole also can be decomposed and expressed as a sum of fractions, and there might be more than one valid decomposition.

Upon still further consideration, students might realize that—

- improper fractions can be composed from the whole part plus one or more units of the fraction part, and
- to compose $\frac{6}{4}$ and $\frac{7}{4}$, the purple rod must be used as the whole.

The ability to compose and decompose numbers flexibly is an important component of a student's number sense. It will serve students well, for example, when they learn to add and subtract unlike fractions.

Extend

Have students think about the following question: *How can you model $1\frac{3}{4}$ if the brown rod represents $\frac{1}{2}$?* Have students give evidence to support their answers. Have them build and sketch models, write sums, and be ready to explain their work.

You can initiate students as follows—

- How would you show the whole, or 1?
- How would you show $\frac{3}{4}$?
- If you start with the whole, what is your first step?
- Does using brown as $\frac{1}{2}$ change the way you write the sum?
- Does it matter if you have different denominators?
- If you wanted to make all the denominators the same, how would you do it?

Look for understanding from students that two brown rods make the whole and that a purple rod represents $\frac{1}{4}$. Look for students who rename fractions when they write their sums.