LESSON 6

Objective

Identify linear equations that have one solution, no solution, and infinitely many solutions.

Common Core State Standards

- **8.EE.7a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Expressions and Equations

One, No, or Infinitely Many Solutions

After achieving proficiency with solving linear equations, students can start to consider specific cases. They are familiar with linear equations with one solution but have not yet encountered equations with no solution or infinitely many solutions. By learning to analyze a solution of the form $x = a$, $a = a$, or $a = b$, where a and b are different numbers, students will be able to determine the number of solutions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How do you solve an equation? What are the steps?* Remind students that the goal is to isolate the variable. Discuss the process with students.
- **Ask:** *What is different about the equations in this problem, compared with the ones you saw when you first started learning to solve equations?*
- **Ask:** *When you solved the second equation, when did you recognize that the equation is an untrue statement? Did you recognize it before you reached 1 = 0 (or 4 = 3)?*
- **Ask:** *When you solved the third equation, when did you recognize that the value of x doesn't matter?*

Solve It

Reread the problem with the students. Have each student use Algebra Tiles™ and the Algebra Tiles Equations Mat to solve all three equations. Compare the solutions. Discuss the forms of the solutions that give one solution, no solution, and infinitely many solutions.

More Ideas

For another way to teach about solutions to equations—

- Solve the equations using Algeblocks® and the Equations Mat.

Formative Assessment

Have students try the following problem.

Henry solved an equation and found it had infinitely many solutions. Which equation did he solve?

A. $14 = 4x + 8$ **B.** $20 = 5x$ **C.** $5(x + 1) = 5x + 5$ **D.** $4(x + 2) = 4x + 9$

Try It! 25 minutes | Pairs

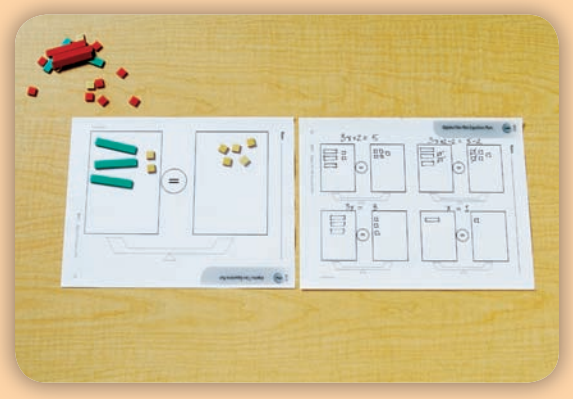
Here is a problem about linear equations with one solution, no solution, and infinitely many solutions.

Kedrick wrote three equations to quiz his brother before a test. He wrote $3x + 2 = 5$, $3x + 4 = 3(x + 1)$, and $3(x + 2) = 3x + 6$. What is the solution to each equation?

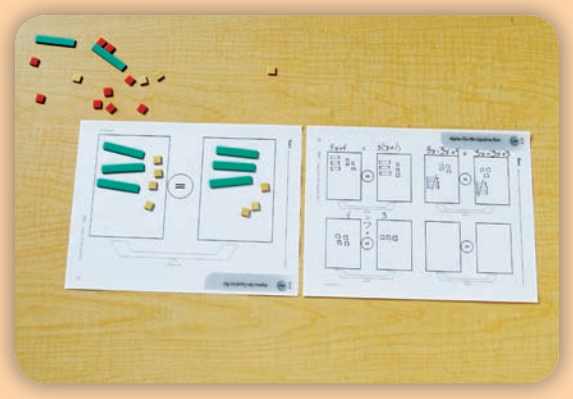
Materials

- Algebra Tiles™
- Algebra Tiles Equations Mat (BLM 2; 1 per pair)
- Algebra Tiles Mini Equations Mats (BLM 3; 3 per student)

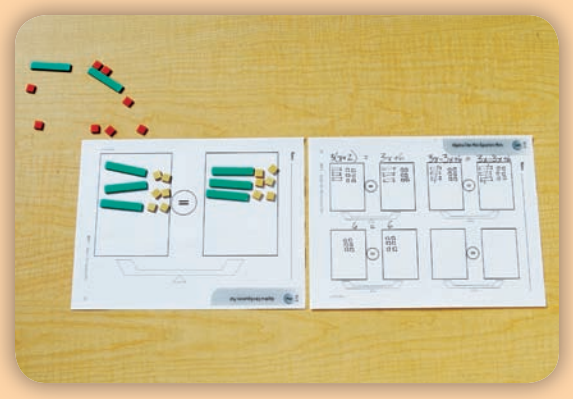
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.



1. Address the first equation. **Ask:** *How will you model the equation?* Elicit from students that they will use a green rectangle for each x and a tan square for each unit. Have them model and solve the equation, recording each step on a Mini Equations Mat. **Ask:** *What is the solution? How many values of* x *satisfy the equation?* Elicit that the solution is $x = 1$. Have students record this result.



2. Have students model and solve the second equation, recording each step on a Mini Equations Mat. **Ask:** *What is the solution? Is anything wrong?* Elicit that the resulting equation, $1 = 0$ (or $4 = 3$), is not a true statement. Explain that this means there are no solutions. Have students record this result.



3. Have students model and solve the third equation, recording each step on a Mini Equations Mat. **Ask:** *What is the solution?* Elicit that the resulting equation, $0 = 0$, is always true. It doesn't matter what you substitute for x in the original equation. Explain that this means there are infinitely many solutions. Have students record this result.

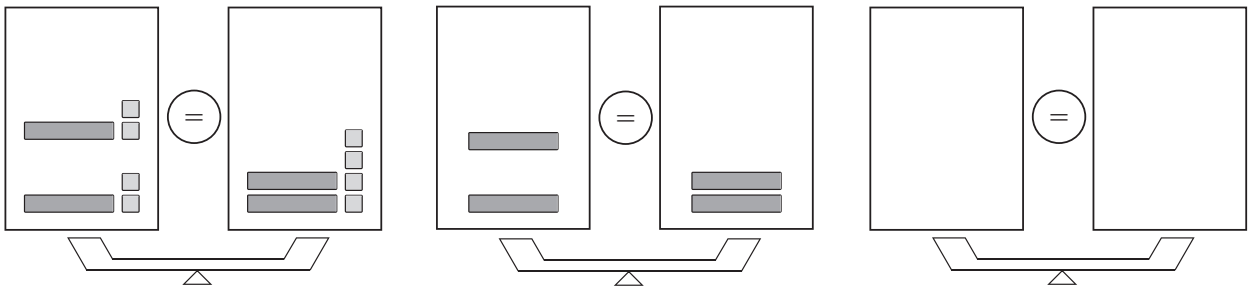
⚠ Look Out!

Students might confuse whether $a = a$ or $a = b$ means infinitely many solutions. For an $a = a$ case, have students go back a step in the solution. Students might feel more comfortable seeing the x in the equation and recognizing the equality in that step.

Answer Key

Use Algebra Tiles and the Algebra Tiles Equations Mat to model the equation shown and then solve it. Write the equation and the solution. (Check students' work.)

1.

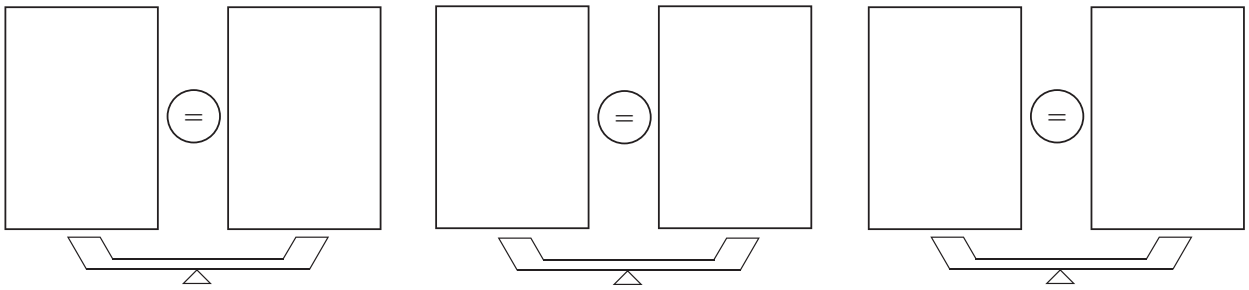


$2(x + 2) = 2x + 4$

infinitely many solutions

Using Algebra Tiles and the Algebra Tiles Equations Mat, model the equation. Sketch the model and solution. Write the solution.

2. $2x + 4 = 12$ $x = 4$ (Check students' models.)



Solve the equation. Show your steps.

3. $3(x + 4) = 3x + 12$

0 = 0, infinitely many solutions

4. $33 = 8x - 7$

$5 = x$

5. $5(x - 1) = 5x - 4$

–5 = –4 or 0 = 1, no solutions

6. $7(x - 6) = 7x + 3$

–42 = 3 or 0 = 45, no solutions

Answer Key

Challenge! Give examples of equations and their solutions. Include equations with 1 solution, no solutions, and infinitely many solutions.

Challenge: Check solutions. Any solution of the form $x = a$ indicates one solution; $a = a$ indicates infinitely many solutions; $a = b$ indicates no solution.