How Using Manipulatives to Teach Math Concepts Aligns with Research

hand2mind®

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Introduction

Whether the abacus was originally invented by the Romans or the Chinese may still be in dispute, but researchers have known for many years that using manipulatives is helpful in teaching math. When using concrete objects to illustrate mathematical principles, students participate more effectively in lessons and problem-solving because being "active" in learning increases students' attention and engagement.[1]

Learning math is difficult for many children. Psychologist Jean Piaget, an early child development theorist, believed that for children to be successful with abstract math they needed to work with models to grasp mathematical concepts.[2] Integrating manipulatives into math lessons and allowing students to be hands-on is referred to as "constructivism"—students are literally constructing learning as they work with concepts and concrete objects. Educator Deborah Ball believes that it is the "context with which the manipulatives are used that creates meaning, such as talk and interaction between teacher and students."[3]

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Over time, many teachers have found the use of concrete objects helpful in teaching new mathematical concepts. Some researchers have hypothesized that using manipulatives for math instruction:

- Reduces students' cognitive load— the amount of working memory required for the activities; and
- Facilitates understanding by anchoring new information in prior knowledge; and
- Increases student motivation to learn and understand.[4]

In a deep meta-analysis of activity-based math learning in grades K–8, researchers determined that student math achievement increased when manipulatives were used (Hartshorn & Boren, 1990).[5] In 1996, researchers Cordova and Lepper found that more engagement with manipulatives led to better learning outcomes. In fact, teaching math concepts with manipulatives is now a core element of all elementary math textbooks.[6]

Using manipulatives for hands-on learning is so effective that the National Council of Teachers of Mathematics (NCTM) has recommended the use of manipulatives in teaching mathematical concepts at all grade levels.[7] Teachers in primary grades have used manipulatives to

1 Retrieved from https://docs.lib.purdue.edu/cgi/viewcontent.cgi?article=1061&context=jps
2 Retrieved from https://nsuworks.nova.edu/cgi/viewcontent.cgi?article=1013&context=transformations/
3 Ibid.
4 Retrieved from https://docs.lib.purdue.edu/cgi/viewcontent.cgi?article=1061&context=jps
5 Retrieved from https://eric.ed.gov/?id=ED321967
6 Retrieved from https://www.tandfonline.com/doi/abs/10.1080/00131720108984802
7 Retrieved from https://eric.ed.gov/?id=ED321967

teach math for many years. However, recent studies have indicated that manipulatives can be effective in helping students learn mathematical concepts across all grades.[8] This is part of the research underpinning that led to the NCTM recommendation to use manipulatives across all grades because it is supported by both learning theory and classroom-based research.

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This white paper explores the research that supports hands-on learning with manipulatives as a research-based instructional strategy for teaching and learning math concepts.

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Benefits of Teaching Math with Manipulatives

Researchers Boggan, Harper, and Whitmore affirm that it is important for children to use manipulatives to sort, classify, weigh, stack, and explore if they are to construct math knowledge (2010). "Children need firsthand experiences related to math, interaction with other children and adults concerning their experiences, and time to reflect on the experiences" say Seefeldt & Wasik.[10]

Seefeldt and Wasik also report that manipulatives can be used to teach a variety of concepts outlined by the NCTM: problem solving, communicating, reasoning, connections, and estimation. They note that using the manipulatives should "foster children's concepts of numbers and operations, patterns, geometry, measurement, data analysis, problem solving, reasoning, connections, and representations."[11]

One example that many teachers are familiar with is teaching fractions using pie pieces. Whether it's a fruit pie or a pizza, the pie effectively communicates that the slice, or fraction, is part of the whole pie. This example illustrates how concrete materials are beneficial when they are able to make aspects of the concept obvious. The context in which the materials are used is also critical. The kind of activities students work through when they are problem solving with manipulatives influences what is learned and where the knowledge can be transferred.[12]

8 Ibid.
9 Retrieved from https://files.eric.ed.gov/fulltext/EJ1096945.pdf
10 Ibid.
11 Ibid.
12 Retrieved from https://docs.lib.purdue.edu/cgi/viewcontent.cgi?article=1061&context=jps

Stein and Bovalino (2001) found that by giving students concrete ways to compare and operate on quantities using manipulatives such as pattern blocks, tiles, and cubes, students could think and reason in more meaningful ways. "The use of manipulatives in the classroom is necessary; it offers a natural way for children to make sense of the mathematics they are trying to learn."[13] Another researcher, Peggy Moch, states that "the relevant application of manipulatives to real-world as well as classroom situations helps students visualize and develop problem-solving strategies."[14]

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Researchers Furner and Worrell (2017) caution that teachers cannot assume that students will automatically discern the correct conclusions from using manipulatives without guidance. While manipulatives are a great tool, students still need help making the connection between the manipulative and the math concept it represents.[15] Since many teachers see manipulatives as appropriate only for the early grades, researcher Dana Freer Weiss wrote about how extending the use of manipulatives benefits middle school students. "Students need guidance to discover the relationship between the concept being taught and the activity with the manipulatives," she writes. "Conceptual understanding enables students to solve problems they have not encountered before."[16]

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By guiding discussions and using appropriate vocabulary as recommended by the NCTM, teachers help students expand their mathematical vocabularies and understanding. "Conceptual approaches to computation instruction result in good achievement, good retention, and a reduction in the amount of time children need to master computational skills," (Moch citing Carroll & Porter, 1997). Moch went on to say that researchers Cain-Caston (1996) and Heuser (2000) found that student reflection on the use of manipulatives in math activities, reduced students' math anxiety and enhanced their mathematical learning.[17]

Moving from the Concrete to the Abstract

Research suggests that the use of manipulatives in math instruction is particularly helpful in getting children to move from concrete to abstract understanding. For place value lessons moving from the concrete to the abstract could include pebbles, bundled straws, base-ten blocks, chip trading, and the abacus. Hilde Howden reminds teachers to pay careful attention to the "bridge" between concrete and abstract. Even with scaffolding, students who can solve a problem at the concrete

13 Retrieved from https://www.tandfonline.com/doi/abs/10.1080/00131720108984802
14 Ibid.
15 Retrieved from https://nsuworks.nova.edu/cgi/viewcontent.cgi?article=1013&context=transformations/
16 Retrieved from http://beduc491.pbworks.com/f/manipulatives+for+middle+school.pdf
17 Retrieved from https://www.tandfonline.com/doi/abs/10.1080/00131720108984802

level, may not be able to solve the same problem at the abstract level. Howden notes that the bridge must be created by careful choices of manipulatives.[18]

For example, The Middle Grades Mathematics Project is an activity-based program that uses manipulatives such as tiles, cubes, geoboards, dice, and counters. Using these materials, students can explore various math challenges by building, drawing, and problem-solving with their peers. In middle or high school algebra, using tiles can help students better make the transition to the abstract level of algebra. Connecting geometry to algebra allows students to apply previous knowledge to new topics. While working with the tiles, students are encouraged to draw pictures and to see mental images which helps them understand an application of abstract math.[19]

While research shows the importance of appropriate teacher-guided lessons in helping students transfer knowledge from the concrete to the abstract, there is also a benefit to letting students follow their own interests while manipulating objects. It seems that they can actually learn more through this activity than when the teacher directs every movement.[20] However, researchers caution that teaching mathematics using manipulatives requires that the teacher has the knowledge, skills, and experience to support students learning math this way.

New brain research reveals the importance of visual thinking to all levels of math. Educator and researcher Jo Boaler asserts there are now multiple studies that demonstrate that "visual mathematics problems help students and raise achievement." She maintains that "strong mathematics learners are those who think deeply, make connections, and visualize.[21] Even though educators have known that using manipulatives helps students understand math concepts, what is new is that "the neurobiological basis of mathematics cognition involves complicated and dynamic communication between the brain systems for memory, control and detection, and the visual processing regions of the brain."[22]

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Working with manipulatives is one way to visualize math and to make connections. Boaler advocates increasing the visual aspects of math to engage more students and help them better understand mathematical ideas. She also notes that in this era of big data, there is growing

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18 Retrieved from https://eric.ed.gov/?id=EJ344475
19 Retrieved from https://eric.ed.gov/?id=ED321967
20 Retrieved from https://www.tandfonline.com/doi/abs/10.1080/00131720108984802
21 Retrieved from https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/04/JACmaths-seeing-article.pdf
22 Ibid.

demand for greater insights into data—particularly by showing data patterns visually.[23]

Researcher Peggy Moch worked with a group of fifth graders using tangrams and centimeter cubes to see if using manipulatives in conjunction with their math lessons would improve their understanding. She reported that the results were better than she had hoped:

"Boys and girls who formerly did not care very much for mathematics were now eager and enthusiastic about participating and learning new ideas in mathematics. The children enjoyed having the opportunity to uncover and think through activities using manipulatives, and they looked forward to future opportunities to investigate other concepts. Manipulatives afforded students an opportunity to touch and feel mathematics-not just to see it or hear it."[24]

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As teachers become adept at incorporating manipulatives into their instruction, they are also changing the learning dynamic in the classroom. By encouraging students to work with manipulatives in an open-ended way, they can apply what they are learning to the real world. "Incorporating manipulatives into math lessons in meaningful ways helps students grasp concepts with greater ease, making teaching most effective," (Boggan, Harper & Whitmire, 2010). Additional studies have shown that students who effectively use manipulatives are more likely to be successful than students who have not used manipulatives.[25]

Learning Math the Hands-on Way: How hand2mind Math Aligns with Research

It is clear from the research that students who use manipulatives to learn math have a deeper and broader understanding of how to apply their learning to new situations. In conjunction with good teaching and regular sessions to engage in hands-on learning experiences—both guided and open-ended—students will develop conceptual understanding of essential math principles. Math competency is essential in a technological world. Math is a critical component of STEM education and the future technology jobs for today's students.

hand2mind has a distinguished history of providing hands-on learning resources for generations of learners. These resources are designed to support the latest research-proven strategies for hands-on math learning. The hand2mind math supplemental programs also support social-emotional learning objectives. Through the hands-on learning process, students develop crucial information about themselves and

23 Ibid.
24 Retrieved from https://www.tandfonline.com/doi/abs/10.1080/00131720108984802
25 Retrieved from https://files.eric.ed.gov/fulltext/EJ1096945.pdf

how to interact appropriately with their peers. The soft skills that they need to learn for future success—communication, collaboration, critical thinking, and creativity are embedded in the learning experiences.

Daily Math Fluency

In just 10 minutes each day, teachers can build math fluency with a year-long supplemental program that provides everything teachers need to teach and reinforce number sense. With specific strategies and manipulatives for visual models, these math talks and number strings improve students' abilities to think about numbers flexibly, efficiently, and accurately. Research-based active learning strategies engage and motivate students and keep them focused on skill development for K–5 math success.

Differentiated Math Centers

Differentiated Math Centers allow teachers to easily incorporate differentiation into their K–5 math instruction. Whether used as a stand-alone learning center or with small groups, these math centers offer multiple ways to engage students in fun and motivating math learning. Each center includes hands-on manipulatives that deepen understanding as students focus on specific standards-based skill development. The differentiation options allow teachers to scaffold learning of each objective through leveled activities, giving all students an opportunity to be successful with core objectives.

Guided Math

Developed by an experienced education specialist, teachers can pick and choose among the individual units or use the entire program for 180 days of instruction. This easy-to-implement, K–5 guided math solution includes detailed lesson plans with hands-on activities that deepen student understanding of core math concepts. Used every day, it is a well-rounded supplemental solution comprised of student-facing learning materials as well as instructional support for teachers. Not only is hands-on learning engaging for students, but it maximizes teachers' effectiveness and efficiencies. All the instructional strategies are research-based.

Math Tasks

Math Tasks Teacher Guides support researcher Peggy Moch's assessment that "the relevant application of manipulatives to real-world as well as classroom situations helps students visualize and develop problem-solving strategies." Using the most common manipulatives to promote multiple solutions, these guides encourage collaboration and help develop K–8 students' deep understanding across standards-based topics and domains.

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Through the Big Idea for each lesson, teachers gain insight into the underlying mathematics of the activity and discover some of the strategies that students are apt to use as they work. Rich problems stimulate many different problem-solving approaches and lead to a variety of solutions. These explorations bring children to new mathematical ideas and deepen their skills.

Hands-On Standards

Teachers can supplement any core math curriculum with these detailed K–8 lesson plans that focus on the most challenging topics for students. The photo-illustrated, step-by-step guides enable teachers to differentiate instruction for intervention, small group, or whole class instruction. Targeted lessons are organized by math standard. The learning outcomes are facilitated by the research-based strategies discussed above—the hands-on lessons help students build conceptual understanding through concrete, representational, and abstract progression.

VersaTiles®

Classroom-proven VersaTiles kits contain all of the benefits of the research cited above for using manipulatives for deeper learning. VersaTiles Math uses purposeful practice so that students grasp all the math skills and concepts required by the math standards for their grade. Teachers can address any math content gaps for their K–8 students and be assured that students can check and correct their own work whether they are working alone or in small groups. The content is aligned to the appropriate standards across all math domains; the visual models made with VersaTiles help develop students' conceptual understanding.

Conclusion

The benefits of using manipulatives to teach mathematical concepts has been extensively researched as laid out in the first part of this white paper. The hands-on learning process deeply engages students and leads to the development of essential math competencies. The hand2mind products have been designed to support standards-based, hands-on learning in well-structured supplemental programs. These age-appropriate programs help teachers differentiate instruction so that all students can master the math standards. The research supports and the products exemplify engaging hands-on learning at its best.

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